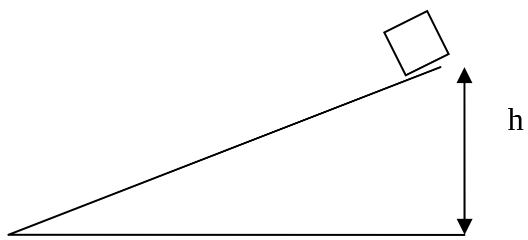
PHYS 101 MIDTERM -- SOLUTIONS October 23rd, 2006

Short answer questions (answer all):
you should not need to do any calculations for these questions. Answer in **a few words, a short phrase, or a simple sketch.**

1) [2 pts] A person is seated on a Ferris wheel (a wheel that rotates in a vertical plane with seats that swivel so that the person is always seated in the head-up position) that rotates at constant speed. At what point around the wheel is the normal force that the seat exerts on the person the largest?

   ANSWER: At the bottom, where the normal force must balance mg downwards AND provide the centripetal force upwards.

2) [2 pts] A block slides down a frictionless inclined plane starting from rest at a vertical height h. (1 pt) Does the speed of the block at the bottom depend on the angle of the slope? (1 pt) Does the acceleration of the block depend on the angle?

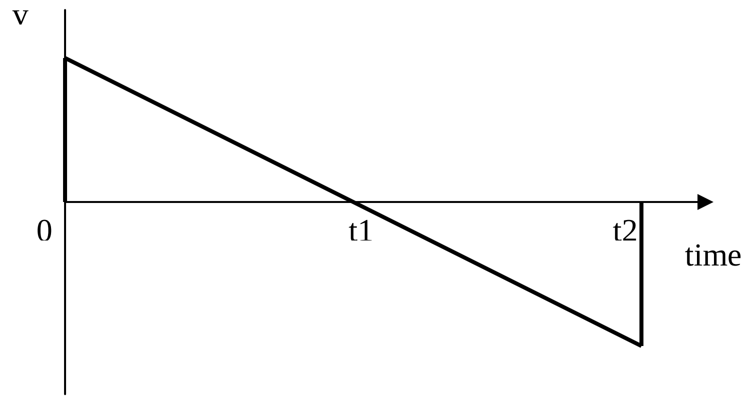


   ANSWER: a) no, by conservation of energy. b) yes, because the force down the incline is mg x sin(θ) where θ is the angle of the incline.

3) [2 pts] Your kid brother is whirling a weight around on a string horizontally over his head. You hear him explain to his friend "the weight stays at the same distance from my hand because the force in the string is balanced by a centrifugal force outwards." Is this a valid explanation? If not, why not?

   ANSWER: No, because there is no outwardly directed "centrifugal" force. There is only one, provided by the string, which keeps the weight moving in the circular path.

4) [2 pts] By looking at the following graph of velocity vs. time, find the time of maximum displacement of the motion. No calculations are necessary.



ANSWER: t1. This is the graph of an object being thrown (or launched) upwards with an initial velocity, reaching the top of its trajectory at t1, and then falling downwards.

5) [2 pts] In the following diagram, a person is either pushing or pulling an object at a constant velocity. The applied force is shown by the arrows. Assuming that there <u>is</u> friction in the system, which action (pushing or pulling) requires the smaller force? Explain briefly.

ANSWER: pulling. The upward component of the force reduces the normal force provided by the surface, and thus the force of friction.

<u>Problems (do 3 out of four):</u>

1) [10 pts] Io is a moon of the planet Jupiter. Io's distance from Jupiter (assume a circular orbit) is 4.22×10^8 m, its mass is 9.03×10^{22} kg, and Jupiter's mass is 1.90×10^{27} kg.

   a) What is Io's speed in its orbit?

   b) What is its period (time for one revolution)?

ANSWER: Io's speed is $2\pi\, r/T$ where r is the radius of the orbit and T is the period. The period (part b) is found from Kepler's third (harmonic) law: $T^2/r^3 = 4\pi^2/(GM)$; G is Newton's constant and M is the mass of JUPITER (the central body). (This equation is given on the formula sheet).

So T = $\underline{1.53 \times 10^5 \text{ sec} = 42.5 \text{ hours}}$

The speed is then 2π r/T = $\underline{1.73 \times 10^4 \text{ m/s} = 17.3 \text{ km/s}}$

2) [10 pts] A rocket is launched at a 60^o angle from the horizontal. The rocket accelerates at 25 m/s^2 for the first 8 seconds, always at the same 60^o angle, as the rocket engine burns. At that point (8 seconds after launch) the fuel is exhausted but the rocket continues to move in projectile motion.

   a. What is the final height the rocket achieves?

   b. What is the final range (ie, how far from the launch point does the rocket touch down)?

   Hint: for the first 8 seconds this is <u>rectilinear motion</u> – the 25 m/s^2 is the total acceleration. You don't even need to consider gravity during this first part. You will need to consider it afterwards, though.

   ANSWER: Note that the first part of the motion is in a STRAIGHT LINE (although it's inclined) – you're given the net acceleration, so you don't have to consider g or anything else. So during this 12 seconds the rocket goes a distance (along the diagonal) of $d=1/2\ at^2$ = 800 m and attains a speed of v=at = 200 m/s. Thus at the end of this 8 seconds, the rocket is at a position of (400,693) m, with a speed of 200 m/s at an angle of 60^o to the horizontal.

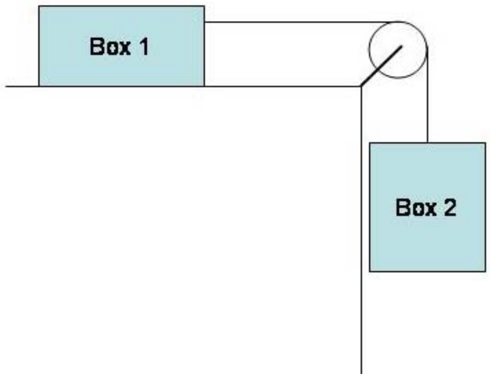
   From there we use projectile motion. The initial y velocity, v_{0y} is 173 m/s so the rocket continues to rise for a time t where $v=0=v_{0y} - gt$, that is for 17.7 seconds. The additional vertical height can be found from $v^2 = v_{0y}^2 + 2ad$, with v=0 (at the top point). We get d=1527 m but we have to add the height during the first 8 seconds, for a total height of <u>2220 m</u>.

   For the final range, calculate the velocity in the y direction when the rocket reaches the ground again: $v_y^2 = v_{0y}^2 - 2gd$. "d" here is -693 m (distance vertically to the ground from the 8 second point). We get v_y = - 208.6 m/s (note the sign, that means that the rocket is moving down). With this vy we use $v_y = v_{0y} + at$ to find the time until touchdown (38.9 seconds). The distance from the burn-out point horizontally is thus 38.9 x 200 cos(60) = 3890 m and to this we have to add the horizontal point at burnout: the total range is <u>4290 m</u>.

3) [10 pts] Two boxes are connected by a cord running over a frictionless pulley as shown below. Box 1 of mass 10.0 kg slides on the top of the table; the coefficient of kinetic friction between it and the table is 0.12. Box 2 has a mass of 15.0 kg.

   a) Draw the free-body diagram for **each** box, identifying all of the forces acting.
   b) Calculate the acceleration of box 2 downwards.

c) Calculate the tension in the cord. (Hint: use the free body diagram for box 2 and the acceleration of the box calculated in b)).



ANSWER: The free body diagram for Box 1 has m_1g (down), F_N (up), F_T to the right and $F_{friction}$ to the left. For Box 2 we have m_2g (down) and F_T (up). The F_T's are the same (in magnitude).

For Box 2 we have the acceleration downwards as $a_2 = (m_2g - F_T)/m_2$. Box 1 has a net force to the right of $F_T – F_f = F_T - \mu m_1g$, so $a = (F_T - \mu m_1g)/m_1$. From each of these two expressions, isolate F_T and then equate them to solve for a. We get: $a = g(m_2 – \mu m_1)/(m_2+m_1)$ which makes perfect sense: it's the net force "down" (m_2g minus the resistive force of friction on box 1) divided by the total mass of the system $m_1 + m_2$.

Numerically, $\underline{a = 5.4\ m/s^2}$.

For the tension in the cord, go back to $a_2 = (m_2g – F_T)/m_2$, or equivalently $F_T = m_2g – m_2a_2 = \underline{66\ N}$.

4) [10 pts] A jet launched from an aircraft carrier is assisted by a catapult. The jet's engines provide a constant thrust of 2.3×10^5 N and the take-off distance is 87 m. At take-off the jet's kinetic energy is 4.5×10^7 J.

a) What is the work done on the jet by the catapult?

b) What is the new take-off distance if the catapult is not used?

ANSWER: The "catapult" is horizontal (like a big spring). The jet's engines provide a work of $Fd = 2.00 \times 10^7$ J. The kinetic energy of the jet KE_{jet} is 4.5×10^7 J so the difference is the work done by the catapult: 2.5×10^7 J.

Without the catapult the total energy must be supplied by the engines, so $d = KE_{jet}/F = \underline{196\ m}$.